Quiz 5

Date: November 21, 2025

Question 1. 5 points

Explain the error in the following proof.

Proposition: The interval $[0, 1]$ is countable.
Proof: Every element of $[0, 1]$ has a decimal expansion $.a_1a_2a_3\ldots$ where $a_i \in \{0, 1, 2, \ldots, 9\}$. So, as there are countably many decimal places and countably (in fact finitely many!) numbers that can go into each decimal place, there can be only countably many such numbers as we proved in class that if X and Y are countable then $X \times Y$ is countable. ■

Question 2. 10 points

Let X_1, X_2, Y_1, and Y_2 be sets. Assume that $\#X_i = \#Y_i$ for $i = 1, 2$. Prove that

$$\#(X_1 \times X_2) = \#(Y_1 \times Y_2).$$